On The Dyadic Green's Function For a Planar Multilayered Dielectric/Magnetic Media

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***Abstract*—A complete plane wave spectral eigenfunction expansion of the electric dyadic Green's function for a planar multilayered dielectric/magnetic media is given in terms of a pair of the ($\hat{z}$)-propagating solenoidal eigenfunctions, where ($\hat{z}$) is normal to the interface, and it is developed via a utilization of the Lorentz reciprocity theorem. This expansion also contains an explicit dyadic delta function term which is required for completeness at the source point. Some useful concepts such as the effective plane wave reflection and transmission coefficients are employed in the present spectral domain eigenfunction expansion. The salient features of this Green's function are also described along with a physical interpretation.**

I. Introduction

A COMPLETE plane wave spectral (PWS) type eigenfunction expansion of the electric dyadic Green's function for the planar multilayered dielectric/magnetic media is given in this paper in terms of a pair of the ($\hat{z}$)-directed solenoidal eigenfunctions, where ($\hat{z}$) is normal to the interface, and it is developed via a utilization of the Lorentz reciprocity theorem. This expansion also contains an explicit dyadic delta function term which is required for making the representation complete at the source point. The geometry of this problem is shown in Fig. 1. The electrical parameters in each of the layers are assumed to be homogeneous and isotropic. It is shown that the field at a given point consists of four distinct wave types (two for each TE and TM type) caused by the presence of the multilayered media. This dyadic Green's function is useful in many problems dealing with the stratified media, i.e., scattering from buried objects in the layered earth, or in the design of high performance finite phased arrays in multilayered dielectric/magnetic environment. Since the dyadic Green's function derived here is for an arbitrarily oriented current point source, it can also be utilized for the applications where the current elements are obliquely rather than horizontally or vertically oriented with respect to the planar interfaces.

The plane wave spectrum (PWS) integral representation of the dyadic Green's function for this canonical problem may be constructed in several ways. One of the most common approaches is to express the Green's function in terms of a magnetic vector potential [1]–[5], whereas another approach is to construct the Green's function from a set of appropriate electric and magnetic vector potentials [6]–[10], [21]. In the former case, the magnetic vector potential in general has components which are parallel and normal to the interface even if the electric point current source does not possess a component which is normal to the interface. In the other approach, the magnetic and electric vector potentials are generally chosen so that they are both normal to the interface. If the electric point current source is chosen normal to the interface, then the two approaches become identical since only a single normally directed magnetic vector potential suffices in this case. This is related to the fact that the normally oriented current moment only excites the TM waves (with respect to the ($\hat{z}$)-coordinate direction), whereas the electric current moment parallel to

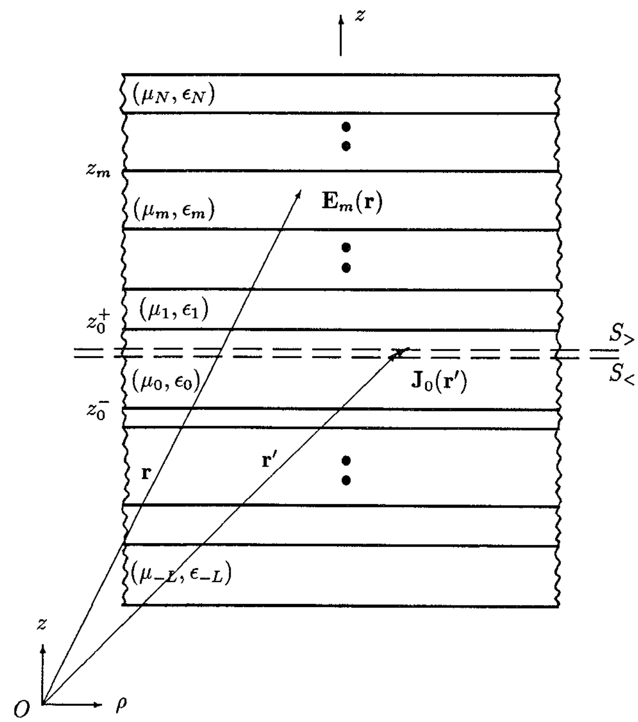

Fig. 1. Electric point current dipole source in a multi-layered dielectric/magnetic media. Also the planar surfaces $S_>$ and $S_<$ slightly above and below the source are shown.

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the interface excites both TM and TE waves. Therefore the total electromagnetic waves must be constructed either with the magnetic vector potential which can produce both TM and TE waves (in this case magnetic vector potential must have components normal and parallel to the interface) in order to satisfy the appropriate boundary conditions, or with the magnetic and electric vector potentials which are both normal to the interface (since a normally directed magnetic vector potential produces TM waves and a normally directed electric vector potential produces TE waves). One of the main advantages of the latter formulation is that the boundary conditions associated with the differential operators for the two different types of vector potentials can be decoupled. In the case of a choice of a single type of magnetic vector potential containing both a vertical ($\hat{z}$) and a horizontal (transverse) to ($\hat{z}$) component, the transverse component (parallel to interface) of that magnetic vector potential will contribute to both TE and TM waves; therefore, the boundary conditions for normal and transverse potential components will be coupled. This disadvantage will be more pronounced if one deals with the stratified or multilayer dielectric/magnetic media, for which the number of coupled boundary conditions increase, thereby complicating the analysis. Recently Bagby and Nyquist [11], derived a formal representation of the dyadic Green's function for the multilayered media in terms of the magnetic vector potential [1], [4], which they specialized for the cases of microstrip and optical circuit structures. Since only the magnetic vector potential is used, the boundary conditions for the TM and TE waves are coupled in [11], hence, the natural distinction between the two is lost. Also the dyadic delta function term, which makes the representation complete at the source point, was not explicitly extracted in [11]; Viola and Nyquist [12], slightly modified that analysis later to properly extract the dyadic delta function term. In the present work, we have derived a complete eigenfunction expansion of the dyadic Green's function for the planar multilayered dielectric/magnetic media using the ($\hat{z}$)-directed solenoidal electric and magnetic (TM and TE) eigenfunctions. We have used continuous eigenmodes propagating along a "preferred" ($\hat{z}$)-direction. We have also employed the orthogonality properties of the eigenmodes over an open planar surface [6] transverse to the direction of the propagation, ($\hat{z}$) to construct our Green's dyadic. This is a generalization of the discrete eigenvalues and eigenmodes, that is usually used in the guided wave theory [13]. Hence, unlike the work reported previously, this analysis retains the connection between the closed (waveguides) and open (planar multilayer) type structure, which is usually lost in the formal Fourier transform method. In addition, because those eigenvalues and eigenmodes are only a function of the geometry of structure, and not the excitation [6], [13], the natural (TM and TE) eigenmodes reveal the physical behavior of the fields in the multi-layered dielectric/magnetic media. Finally, we have employed a method that utilizes only the solenoidal eigenfunctions [14], and hence, the dyadic delta function term at the source point is included explicitly as a correction to the general solenoidal eigenfunction expansion which is valid outside the source point. The electric dyadic Green's components given in this work appear to be closely related to those electric field components which have been derived by Kong [7], [8], and Chew [21] utilizing the usual boundary conditions at each of the interfaces and the proper condition at the source point. As indicated above, the procedure used here is somewhat different, in that we have utilized the orthogonality of continuous eigenmodes at the planar interfaces along with the Lorentz reciprocity theorem to drive the complete eigenfunction expansion of the electric dyadic Green's function which contains a physical interpretation.

The format of the paper is as follows. In Section II, we outline the procedure required to derive the complete eigenfunction expansion of the dyadic Green's function for the multilayered media, $\mathcal{G}^{m,0}$, in terms of only the solenoidal eigenfunctions. In Section III, we start with the unbounded case, in which the point source radiates with no interface present, and construct the corresponding dyadic Green's function, $\mathcal{G}^0$, in terms of an integral over the spectra of plane waves that constitute the continuous eigenfunction expansion in which the eigenfunctions are guided in the preferred $\hat{z}$-coordinate direction, using the procedure described in Section II. This is essentially the z-propagation (plane wave spectrum) representation of the free space dyadic Green's function which is usually represented by the discrete spherical vector wave type radially propagating eigenfunction expansion. In Section IV, the dyadic Green's function for the multilayered media, $\mathcal{G}^{m,0}$, is then constructed from the principle of the superposition, which involves the sum of the fields of firstly the source in free space (or the free space Green's function $\mathcal{G}^0$) and secondly the fields scattered by the layered media. Section V deals with the physical interpretation of the dyadic Green's function and numerical results. Conclusions and discussions are presented in Section VI.

II. Formulation of $\mathcal{G}^{m,0}$ in Terms of the Solenoidal Eigenfunctions

In this section we outline a general procedure described by Pathak, [14], which can also be employed to find a complete eigenfunction expansion of the electric field in the multilayered media, $\boldsymbol{E}_m$, and its corresponding dyadic analog $\mathcal{G}^{m,0}$ in terms of only the solenoidal eigenfunctions.

The usual Maxwell curl equations for the electric and magnetic fields $\boldsymbol{E}_m$ and $\boldsymbol{H}_m$ within any mth layer (see Fig. 1), respectively, are given by

$$\nabla \times \boldsymbol{E}_m = -j\omega\mu_m \boldsymbol{H}_m; \quad \nabla \times \boldsymbol{H}_m = j\omega\epsilon_m \boldsymbol{E}_m + \boldsymbol{J}_0. \quad (1)$$

An $e^{j\omega t}$ time dependence is assumed and suppressed in (1), and as usual, μ_m and ϵ_m are the permeability and permittivity of the medium (m), and $\boldsymbol{J}_0$ is the impressed electric current source. If the electric current density $\boldsymbol{J}_0$ is taken to be a point source of strength p_e at $r = r'$ in the region (0); then,

$$J_0(r) \equiv p_e\, \delta(r - r'). \tag{2}$$

Before proceeding further, it is important to relate the dyadic Green's function to the electric field due to J_0 as [15]

$$E_m(r) = -j\omega\mu_0 \iiint_v \mathcal{G}^{m,0}(r, r'') \cdot J_0(r'')\, dv'', \tag{3}$$

where $\mathcal{G}^{m,0}$ is the multilayered electric dyadic Green's function, and v contains the source region. If $J_0(r'')$ is an arbitrarily oriented point current source of the strength p_e given in (2), then electric field may be viewed as a distribution; namely,

$$E_m(r) = -j\omega\mu_0 \mathcal{G}^{m,0}(r, r') \cdot p_e. \tag{4}$$

Let the solenoidal part of the eigenfunction expansion of the electric field E_m, which is valid for $z \neq z'$ (and hence for $r \neq r'$), be denoted by E'_m. The field E'_m is obtained in terms of only the solenoidal eigenfunctions because the electric field has zero divergence for $z \neq z'$. The z-propagating solenoidal eigenfunction expansion of E'_m can be expressed as

$$E'_m = \begin{cases} E_m^>, & z > z' \\ E_m^<, & z < z'. \end{cases} \tag{5}$$

Alternatively, E'_m in (5) can be written as

$$E'_m = \mathcal{U}(z - z')E_m^> + \mathcal{U}(z' - z)E_m^<, \tag{6}$$

where the Heaviside unit step function $\mathcal{U}(\xi)$ is defined by,

$$\mathcal{U}(\xi) = \begin{cases} 1, & \xi > 0 \\ 0, & \xi < 0 \end{cases},$$

and $\gtrless$ means the fields for $z \gtrless z'$. The entire space consists of two regions $z > z'$ and $z < z'$; $z = z'$ is the plane S (normal to $\hat{z}$-axis) containing the source, $J_0 = p_e\, \delta(\rho - \rho')\, \delta(z - z')$, in region (0) of Fig. 1. It is noted that (μ_0, ϵ_0) correspond to the constitutive parameters of the medium in region (0); in general, (μ_0, ϵ_0) are different from those for free space. Consider next the magnetic field H_m due to J_0; in particular making use of (5), yields

$$\nabla \times E_m^\gtrless = -j\omega\mu_m H_m^\gtrless; \quad \nabla \times H_m^\gtrless = j\omega\epsilon_m E_m^\gtrless, \tag{7}$$

where $H_m^\gtrless$ is the value of the magnetic field H_m in the region (m), for $z \gtrless z'$. It is clear from (7) that the magnetic field $H_m^\gtrless$ is known once $E_m^\gtrless$ is known. The fields $H_m^>$ and $H_m^<$ must satisfy the proper source condition at $r = r'$. In order to impose the boundary condition at the source point, $r = r'$, the volume current density J_0 must be expressed in terms of a distribution p_{es} corresponding to a "surface" current density at $z = z'$ (i.e., on the surface S); thus

$$J_0 = p_e\, \delta(\rho - \rho')\, \delta(z - z') = P_e\, \delta(r - r') = p_{es}\, \delta(z - z'). \tag{8}$$

Now the discontinuity of the tangential magnetic field in the region (0), across S (at $z = z'$) must be equal to the surface current density at S; namely,

$$\hat{z} \times (H_0^> - H_0^<) = \hat{I}_t \cdot p_{es}, \tag{9}$$

where $\hat{I}_t$ denotes the transverse part of the unit dyad with respect to $\hat{z}$,

$$\hat{I} = \hat{I}_t + \hat{z}\hat{z}; \quad \hat{I}_t = \hat{x}\hat{x} + \hat{y}\hat{y}. \tag{10}$$

It is clear that (9) is valid only at $z = z'$, so it can be expressed as

$$\hat{z} \times (H_0^> - H_0^<)\, \delta(z - z') = \hat{I}_t \cdot p_{es}\, \delta(z - z'), \tag{11}$$

it follows directly from (8) that the above equation becomes

$$\hat{z} \times (H_0^> - H_0^<)\, \delta(z - z') = \hat{I}_t \cdot p_e\, \delta(r - r'),$$

or more generally,

$$\hat{z} \times (H_m^> - H_m^<)\, \delta(z - z') = \hat{I}_t \cdot p_e\, \delta(r - r'). \tag{12}$$

This is the expression for the condition on H_m at the source point, and it directly indicates the appropriate addition to E'_m at the source point which is required to yield the complete expansion of E_m. It is important to note that since the discontinuity condition in (12) relates $H_m^>$ to $H_m^<$ across the source point, one only needs to know $H_m^>$ and $H_m^<$ to completely specify H_m due to the source $J_0 = p_e\, \delta(r - r')$; thus

$$H_m = \mathcal{U}(z - z')H_m^> + \mathcal{U}(z' - z)H_m^<. \tag{13}$$

The E'_m of (6) can now be readily found by employing (7), and using the relation based on distribution theory, [16],

$$\nabla \times [H_m^\gtrless \mathcal{U}(\pm z \mp z')] = \mathcal{U}(\pm z \mp z')\nabla \times H_m^\gtrless \pm \hat{z} \times H_m^\gtrless\, \delta(z - z'). \tag{14}$$

From (7), (12) and (14), if follows that

$$E'_m = \frac{1}{j\omega\epsilon_m} \nabla \times H_m - \frac{1}{j\omega\epsilon_0} \hat{I}_t \cdot p_e\, \delta(r - r'). \tag{15}$$

The precise relationship between the complete field E_m of (4) and the incomplete field E'_m of (15) can now be written by using (1) and (10), [14],

$$E_m(r, r') = E'_m(r, r') - j\omega\mu_0 \left[-\frac{\hat{z}\hat{z}}{k_0^2}\, \delta(r - r') \right] \cdot p_e;$$

$$E_{m|z \neq z'} = E'_{m|z \neq z'}. \tag{16}$$

The Green's dyadic $\mathcal{G}^{m,0}$ can be inferred from (16) by comparison with (4). Thus,

$$\mathcal{G}^{m,0}(r, r') = \bar{\bar{g}}^{m,0}(r, r') - \frac{\hat{z}\hat{z}}{k_0^2}\, \delta(r - r'), \tag{17}$$

with

$$E'_m(r, r') \equiv -j\omega\mu_0 \overline{\overline{g}}^{m,0}(r, r') \cdot p_e. \quad (18)$$

From the above discussion, it is clear that one can also construct the complete free space dyadic Green's function, $\mathcal{G}^0$, in terms of the "z-propagating" solenoidal eigenfunctions which will be obtained in the following section.

III. Construction of the Free Space Dyadic Green's Function, $\mathcal{G}^0$

In this section, the procedure outlined in the previous section is applied to obtain an explicit expansion for $\mathcal{G}^0$ which is associated with an electric point current source, $J_0 = p_e\, \delta(r - r')$, which radiates in an unbounded medium with parameter (μ_0, ϵ_0) which are the same as in region (0) with no interface present. In the following section, the procedure developed here will be extended to explicitly obtain the dyadic Green's function $\mathcal{G}^{m,0}$ of (17) for the multilayered media. The first step in the procedure for obtaining the free space electric field E_0 and its corresponding $\mathcal{G}^0$ involves the construction of a z-propagating PWS solenoidal eigenfunction expansion of E'_0 which is complete if $z \neq z'$.

The geometry of the problem dealing with a homogeneous (free) space with constitutive parameters (μ_0, ϵ_0) excited by $J_0 = p_e\, \delta(r - r')$ is illustrated in Fig. 2. The solenoidal eigenfunctions for this problem are chosen to propagate in the preferred $\pm\hat{z}$-coordinate direction. The source point at $r = r'$ lies in the plane S at $z = z'$ as in Fig. 2. Let $E^{\gtrless}$ and $H^{\gtrless}$ denote the continuous PWS solenoidal vector wave function expansions for the electric and magnetic fields, due to J_0 in the absence of the interface; thus,

$$E^{\gtrless} = E'^{\gtrless} + E''^{\gtrless}; \quad H^{\gtrless} = H'^{\gtrless} + H''^{\gtrless}, \quad (19)$$

and [6],

$$E^{\gtrless} = \int dk_t\, (a'^{\gtrless} e'^{\gtrless} + a''^{\gtrless} e''^{\gtrless});$$

$$H^{\gtrless} = \int dk_t\, (a'^{\gtrless} h'^{\gtrless} + a''^{\gtrless} h''^{\gtrless}), \quad (20)$$

where prime (′) and double prime (″) refer to TM and TE wave components with respect to the preferred $\hat{z}$-coordinate direction, respectively, and h' and e'' can be derived from the solenoidal magnetic and electric $\hat{z}$-directed vector potentials, (Π', Π''), respectively [6], [13],

$$h'^{\gtrless} = \mp\hat{z} \times \nabla_t \Pi'^{\gtrless};$$

$$e'^{\gtrless} = \pm\frac{1}{j\omega\epsilon_0} \nabla_t \frac{\partial}{\partial z} \Pi'^{\gtrless} \mp \frac{1}{j\omega\epsilon_0} \hat{z} \nabla_t^2 \Pi'^{\gtrless}, \quad (21)$$

and

$$e''^{\gtrless} = \hat{z} \times \nabla_t \Pi''^{\gtrless};$$

$$h''^{\gtrless} = \frac{1}{j\omega\mu_0} \nabla_t \frac{\partial}{\partial z} \Pi''^{\gtrless} - \frac{1}{j\omega\mu_0} \hat{z} \nabla_t^2 \Pi''^{\gtrless}, \quad (22)$$

where, ∇_t is the transverse (to $\hat{z}$) part of the ∇ operator. Electric and magnetic potentials, Π' and Π'', which can also be viewed as a pair of Debye potentials [5], [17] satisfy the well known Helmholtz equation; their associated $\hat{z}$-propagating eigenfunctions can be expressed as

$$\Pi^{\gtrless}(k_t) = \frac{1}{2\pi} \exp[-j(k_t \cdot \overline{\rho} \pm \kappa_0 z)];$$

$$\Pi^{\gtrless}(-k_t) = \frac{1}{2\pi} \exp[-j(-k_t \cdot \overline{\rho} \pm \kappa_0 z)]; \quad (23)$$

where prime (′) and double prime (″) have been omitted for convenience; k_t, κ_0, and $\overline{\rho}$ are respectively defined as

$$k_t = \hat{x}k_x + \hat{y}k_y; \quad k_t = \sqrt{k_t \cdot k_t}; \quad \kappa_0 = \sqrt{k_0^2 - k_t^2}, \quad (24)$$

and

$$\overline{\rho} = \hat{x}x + \hat{y}y; \quad r = \overline{\rho} + \hat{z}z; \quad k_0^2 = \omega^2\mu_0\epsilon_0. \quad (25)$$

In the above formulation, the variable k_t (i.e., $\hat{x}k_x + \hat{y}k_y$; $dk_t = dk_x\, dk_y$) are the continuous eigenvalues which span over the entire spectral domain ($-\infty < k_x < \infty$; and $-\infty < k_y < \infty$). The unknown spectral amplitudes $a'^{\gtrless}$ and $a'^{\gtrless}$ of (20) associated with the TM and TE modal fields respectively, are found from an application of the Lorentz reciprocity theorem to the pair of the fields $(E^{\gtrless}, H^{\gtrless})$ of (20) and the source free solenoidal vector wavefunctions $(e^{\lessgtr}, h^{\lessgtr})$ in the region V_0, bounded by planar surfaces $S_>$ and $S_<$, which are slightly above and below the surface S of Fig. 2, respectively, [13], [14][1]:

$$\iint_{S_> + S_<} ds \cdot (E^{\gtrless} \times h^{\lessgtr} - e^{\lessgtr} \times H^{\gtrless})$$

$$= \iiint_{V_0} dv\, e^{\lessgtr} \cdot J_0$$

$$= e^{\lessgtr} \cdot p_e. \quad (26)$$

The solenoidal vector wavefunctions $e^{\gtrless}$ and $h^{\gtrless}$ satisfy the orthogonality condition on the surface $S_<$ and $S_>$; namely,

$$\iint_{S_{\gtrless}} ds \cdot (e^{<}(\pm k_t) \times h^{>}(\mp k'_t))$$

$$= \iint_{S_{\gtrless}} ds \cdot (-e^{>}(\pm k_t) \times h^{<}(\mp k'_t))$$

$$= (\hat{s}_{\gtrless} \cdot \hat{z})\Omega\, \delta(k_t - k'_t), \quad (27)$$

where Ω can be Ω' or Ω'' for the TM and TE cases, respectively; thus,

$$\Omega' = k_t^2 \eta'_0; \quad \Omega'' = \frac{k_t^2}{\eta''_0}, \quad (28)$$

[1] We apply the Lorentz reciprocity theorem to the volume V_0 here with the radiation condition implied as $\rho \to \infty$.

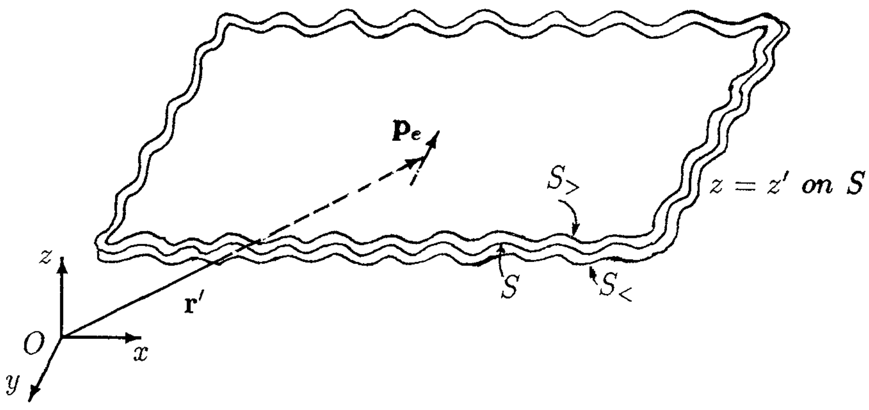


Fig. 2. Imaginary plane S, parallel to xy plane, passing through the source at $z = z'$ in the free space. Also plotted are planar surfaces $S_>$ and $S_<$ slightly above and below the source.

with the unit vector $\hat{s}_{\gtrless}$ directed along the outward normal to the surface $S_{\gtrless} = \pm z$; and η_0' and η_0'' are associated with the TM and TE wave impedances for region (0) and defined as

$$\eta_0' = \frac{\kappa_0}{\omega\epsilon_0}; \quad \eta_0'' = \frac{\omega\mu_0}{\kappa_0}. \tag{29}$$

Incorporating (20) and (27) into (26) yields

$$a^{\gtrless} = \frac{e^{\lessgtr}(-k_t, r') \cdot p_e}{-2\Omega}; \quad r' = \bar{\rho}' + \hat{z}z'. \tag{30}$$

In deriving the orthogonality relationship of (27), use has been made of

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\zeta e^{-j\zeta(\xi - \xi')} = \delta(\xi - \xi'). \tag{31}$$

Therefore, from (30), (20), (16), (6), and (4), the z-propagation PWS representation of the free space dyadic Green's function can be identified as

$$\begin{aligned} \mathcal{G}^0(r, r') = {} & \frac{\mathcal{U}(z - z')}{-j\omega\mu_0}\int dk_t \left(\frac{e'^{>}(k_t, r)e'^{<}(-k_t, r')}{-2\Omega'}\right. \\ & \left. + \frac{e''^{>}(k_t, r)e''^{<}(-k_t, r')}{-2\Omega''}\right) \\ & + \frac{\mathcal{U}(z' - z)}{-j\omega\mu_0}\int dk_t \left(\frac{e'^{<}(k_t, r)e'^{>}(-k_t, r')}{-2\Omega'}\right. \\ & \left. + \frac{e''^{<}(k_t, r)e''^{>}(-k_t, r')}{-2\Omega''}\right) - \frac{\hat{z}\hat{z}}{k_0^2}\delta(r - r'). \end{aligned} \tag{32}$$

The PWS for the fields (E_0 and H_0) due to p_e in free space, and hence for the corresponding free space dyadic Green's function $\mathcal{G}^0$ given above provides information on the general form of the PWS solution for the fields E_m and H_m for the multilayered case and therefore also on the dyadic Green's function for the multilayered media, which will be discussed in the following section.

IV. Construction of the Multilayered Dyadic Green's Function, $\mathcal{G}^{m,0}$

The electric dyadic Green's function for the multilayered media can be expressed as a sum of $\mathcal{G}^0$ in (32) and another contribution to account for the field scattered by the layered media. The scattered contribution can be expressed in terms of a PWS integral resembling that for $\mathcal{G}^0$. Let us consider an arbitrarily oriented point dipole source in a general multilayered media with constitutive parameters μ_m and ϵ_m, as shown in Fig. 1. The source is located in region (0) with constitutive parameters μ_0 and ϵ_0. In order to find the explicit value of the fields in each region, one can write the field quantities as the superposition of the four traveling waves (two oppositely traveling waves for each mode) with unknown coefficients and then solve for the unknown coefficients by enforcing the continuity of tangential electromagnetic fields quantities at each interface, [7], [8], [21]. However, we pursue another approach, which provides a useful physical interpretation for the dyadic Green's function. From (21) and (22), one can see that the continuity of the tangential field quantities at the interface m imply

$$\Pi'_{m-1} = \Pi'_m; \quad \frac{1}{\epsilon_{m-1}}\frac{\partial}{\partial z}\Pi'_{m-1} = \frac{1}{\epsilon_m}\frac{\partial}{\partial z}\Pi'_m, \tag{33}$$

and

$$\Pi''_{m-1} = \Pi''_m; \quad \frac{1}{\mu_{m-1}}\frac{\partial}{\partial z}\Pi''_{m-1} = \frac{1}{\mu_m}\frac{\partial}{\partial z}\Pi''_m. \tag{34}$$

These boundary conditions are analogous to the continuity of the current and voltage at each discontinuity of a piecewise uniform transmission line for which the characteristic impedance (and the wave number) is defined in each layer as, [6], [9]

$$\eta_m' = \frac{\kappa_m}{\omega\epsilon_m}; \quad \eta_m'' = \frac{\omega\mu_m}{\kappa_m}, \tag{35}$$

where prime (′) and double prime (″) are associated with TM_z and TE_z cases respectively, and $\kappa_m = \sqrt{k_m^2 - k_t^2}$, is the wave number in the $\hat{z}$-direction.

The field quantities in region m can be expressed as a superposition of known continuous solenoidal eigenfunctions that propagate in $\pm\hat{z}$-direction with the unknown spectral weights, a_m, [9]

$$\begin{aligned} \boldsymbol{E}_m^{\gtrless} &= \int dk_t\, (a_m'^{\gtrless}(\boldsymbol{e}_m'^{\gtrless} + R_m'^{\gtrless}(0)\boldsymbol{e}_m'^{\lessgtr}) \\ &\quad + a_m''^{\gtrless}(\boldsymbol{e}_m''^{\gtrless} + R_m''^{\gtrless}(0)\boldsymbol{e}_m''^{\lessgtr})), \\ \boldsymbol{H}_m^{\gtrless} &= \int dk_t\, (a_m'^{\gtrless}(\boldsymbol{h}_m'^{\gtrless} + R_m'^{\gtrless}(0)\boldsymbol{h}_m'^{\lessgtr}) \\ &\quad + a_m''^{\gtrless}(\boldsymbol{h}_m''^{\gtrless} + R_m''^{\gtrless}(0)\boldsymbol{h}_m''^{\lessgtr})), \end{aligned} \tag{36}$$

where $R_m^{\gtrless}(0) = R_m^{\gtrless} e^{\mp j2\kappa_m z_m}$, $R_m'^{\gtrless}$ and $R_m''^{\gtrless}$ are the TM, and TE effective reflection coefficients at the interfaces $(m, m+1)$ and $(m, m-1)$ for $(>)$ and $(<)$, respectively [7], [8], [18], [19], [21]. As discussed in the Appendix, the effective reflection coefficient $R_m^{\gtrless}$ for region m, is a function of reflection coefficients of all successive layers, (i.e., $m \pm 1$, $m \pm 2$, $m \pm 3$, $\cdots$,; $(\pm^{\text{for}\,>}_{\text{for}\,<})$) of the multilayered media, (in particular see (A13) and (A14)). Also the modal coefficients $a_m^{\gtrless}$ of region m, and $a_n^{\gtrless}$ of region n on either side of the source are related via the effective transmission coefficient, $\Upsilon_{m,n}^{\gtrless}$, as is evident from the piece-wise transmission line theory discussed in the Appendix. In view of (A11), (A12), (A16), (21) and (22), one will have[2]

$$a_m^{\gtrless} = e^{\mp j(\kappa_n z_n - \kappa_m z_m)}\Theta_{m,n}^{\gtrless} a_n^{\gtrless};$$

$$\text{where} \begin{cases} \Theta_{m,n}'^{\gtrless} = \dfrac{\eta_{n'}}{\eta_{m'}} \Upsilon_{m,n}'^{\gtrless}, \\ \qquad \text{for TM (′) case} \\ \Theta_{m,n}''^{\gtrless} = \Upsilon_{m,n}''^{\gtrless}, \\ \qquad \text{for TE (″) case.} \end{cases} \tag{37}$$

Hence, one only needs to find the modal coefficients $a_0'^{\gtrless}$, and $a_0''^{\gtrless}$ in region 0, in order to completely specify the fields in all regions. Specifying (36) for region 0 (i.e., $m = 0$) and invoking the Lorentz reciprocity theorem to the pair of fields $(\boldsymbol{E}_0^{\gtrless}, \boldsymbol{H}_0^{\gtrless})$ and a set of source free test fields $(\mathcal{E}_0^{\gtrless}, \mathcal{H}_0^{\gtrless})$ in the region V_0, bounded by the planar surfaces of $S_<$ and $S_>$, slightly below and above the source respectively as shown in Fig. (1), we get

$$\begin{aligned} \iint_{S_> + S_<} ds \cdot (\boldsymbol{E}_0^{\gtrless} \times \mathcal{H}_0^{\lessgtr} - \mathcal{E}_0^{\lessgtr} \times \boldsymbol{H}_0^{\gtrless}) &= \iiint_{V_0} dv\, \mathcal{E}_0^{\lessgtr} \cdot \boldsymbol{J}_0 \\ &= \mathcal{E}_0^{\lessgtr} \cdot \boldsymbol{p}_e. \end{aligned} \tag{38}$$

[2]Note that $\Upsilon_{m,n}^{\gtrless}$ is the ratio of the incident tangential electrical fields of regions m and n; namely,

$$a_m^{\gtrless}(\hat{z} \times \boldsymbol{e}_m^{\gtrless}) = e^{\mp j(\kappa_n z_n - \kappa_m z_m)}\Upsilon_{m,n}^{\gtrless} a_n^{\gtrless}(\hat{z} \times \boldsymbol{e}_n^{\gtrless}).$$

where $(\mathcal{E}_0^{\gtrless}, \mathcal{H}_0^{\gtrless})$ are given by

$$\mathcal{E}_0^{\gtrless} = \boldsymbol{e}_0^{\gtrless} + R_0^{\gtrless}(0)\boldsymbol{e}_0^{\lessgtr};\quad \mathcal{H}_0^{\gtrless} = \boldsymbol{h}_0^{\gtrless} + R_0^{\gtrless}(0)\boldsymbol{h}_0^{\lessgtr},$$

and

$$R_0^{\gtrless}(0) = R_0^{\gtrless}(z_0^{\mp})e^{\pm j2\kappa_0 z_0^{\pm}}. \tag{39}$$

The solenoidal vector functions $(\mathcal{E}_0^{\gtrless}, \mathcal{H}_0^{\gtrless})$ satisfy the orthogonality relationship on the planar surface of $S_{\gtrless}$ as is evident from (27),

$$\begin{aligned} \iint_{S_{\gtrless}} ds \cdot (\mathcal{E}_0^{<}(\pm\boldsymbol{k}_t) \times \mathcal{H}_0^{>}(\mp\boldsymbol{k}_t') - \mathcal{E}_0^{>}(\mp\boldsymbol{k}_t) \\ \times \mathcal{H}_0^{<}(\pm\boldsymbol{k}_t')) = (\hat{s}_{\gtrless} \cdot \hat{z})2\Lambda\, \delta(\boldsymbol{k}_t - \boldsymbol{k}_t'), \end{aligned} \tag{40}$$

with

$$\Lambda = \Omega(1 - R_0^{>}(z_0^{+})R_0^{<}(z_0^{-})e^{-j2\kappa_0 d_0}), \tag{41}$$

where $d_0 = z_0^+ - z_0^-$ is the thickness of the slab 0; z_0^+ and z_0^- are the values of z at the interfaces of region (0) and they are specified in Fig. 1, and Ω is given in (28). Incorporating (36) and (40) into (38) yields

$$a_0^{\gtrless} = \frac{\mathcal{E}_0^{\lessgtr}(-\boldsymbol{k}_t, \boldsymbol{r}') \cdot \boldsymbol{p}_e}{-2\Lambda}. \tag{42}$$

The prime (′) and double prime (″) have been omitted for convenience in (37)–(42). Hence, the electric dyadic Green's function for the multilayered media, $\mathcal{G}^{m,0}$, can be written via (42), (39), (37), (36), (16) and (4) as

$$\begin{aligned} \mathcal{G}^{m,0}(\boldsymbol{r}, \boldsymbol{r}') &= \frac{\mathcal{U}(z - z')}{-j\omega\mu_0} \int dk_t \\ &\quad \cdot \left(\Theta_{m,0}'^{>} \frac{\mathcal{E}_m'^{>}(\boldsymbol{k}_t, \boldsymbol{r})\mathcal{E}_0'^{<}(-\boldsymbol{k}_t, \boldsymbol{r}')}{-2\Lambda'} + \Theta_{m,0}''^{>} \frac{\mathcal{E}_m''^{>}(\boldsymbol{k}_t, \boldsymbol{r})\mathcal{E}_0''^{<}(-\boldsymbol{k}_t, \boldsymbol{r}')}{-2\Lambda''}\right) \\ &\quad + \frac{\mathcal{U}(z' - z)}{-j\omega\mu_0} \int dk_t \\ &\quad \cdot \left(\Theta_{m,0}'^{<} \frac{\mathcal{E}_m'^{<}(\boldsymbol{k}_t, \boldsymbol{r})\mathcal{E}_0'^{>}(-\boldsymbol{k}_t, \boldsymbol{r}')}{-2\Lambda'} + \Theta_{m,0}''^{<} \frac{\mathcal{E}_m''^{<}(\boldsymbol{k}_t, \boldsymbol{r})\mathcal{E}_0''^{>}(-\boldsymbol{k}_t, \boldsymbol{r}')}{-2\Lambda''}\right) \\ &\quad - \frac{\hat{z}\hat{z}}{k_0^2}\delta(\boldsymbol{r} - \boldsymbol{r}'). \end{aligned} \tag{43}$$

where $(\mathcal{E}_m^{\gtrless}, \mathcal{H}_m^{\gtrless})$ are given by $(\mathcal{E}_m^{\gtrless} = \boldsymbol{e}_m^{\gtrless} + R_m^{\gtrless}(0)\boldsymbol{e}_m^{\lessgtr}$; $\mathcal{H}_m^{\gtrless} = \boldsymbol{h}_m^{\gtrless} + R_m^{\gtrless}(0)\boldsymbol{h}_m^{\lessgtr})$. The above expression for $\mathcal{G}^{m,0}$ can be written more explicitly using (39), (37) and (21)–(23) as

$$\begin{aligned} \mathcal{G}^{m,0} &= -\frac{\hat{z}\hat{z}}{k_0^2}\delta(\boldsymbol{r} - \boldsymbol{r}') + \mathcal{U}(z - z')\frac{-j}{4\pi^2}\int dk_t \\ &\quad \cdot e^{j\kappa_m z_m} e^{-j\kappa_0 z_0^+} \frac{1}{2\kappa_0}[\Delta'' \Upsilon_{m,0}''^{>} \hat{\boldsymbol{n}}'' \hat{\boldsymbol{n}}'' \\ &\quad \cdot (e^{-j\boldsymbol{k}_m^{>} \cdot \boldsymbol{r}} + R_m''^{>} e^{-j2\kappa_m z_m} e^{-j\boldsymbol{k}_m^{<} \cdot \boldsymbol{r}}) \end{aligned}$$

$$\cdot\,(e^{+jk_0^> \cdot r'} + R_0''^< e^{+j2\kappa_0 z_0^-} e^{+jk_0^< \cdot r'})$$
$$+\,\Delta' \Upsilon_{m,0}'^> \frac{k_m \kappa_0}{k_0 \kappa_m} (\hat{n}_m'^> e^{-jk_m^> \cdot r}$$
$$+\,R_m'^> e^{-j2\kappa_m z_m} \hat{n}_m'^< e^{-jk_m^< \cdot r})(\hat{n}_0'^> e^{+jk_0^> \cdot r'}$$
$$+\,R_0'^< e^{+j2\kappa_0 z_0^-} \hat{n}_0'^< e^{+jk_0^< \cdot r'})]$$
$$+\,\mathcal{U}(z' - z) \cdot \frac{-j}{4\pi^2} \int dk_t$$
$$\cdot\, e^{-j\kappa_m z_m} e^{j\kappa_0 z_0^-} \frac{1}{2\kappa_0} [\Delta'' \Upsilon_{m,0}''^< \hat{n}'' \hat{n}''$$
$$\cdot\,(e^{-jk_m^< \cdot r} + R_m''^< e^{+j2\kappa_m z_m} e^{-jk_m^> \cdot r})$$
$$\cdot\,(e^{+jk_0^< \cdot r'} + R_0''^> e^{-j2\kappa_0 z_0^+} e^{+jk_0^> \cdot r'})$$
$$+\,\Delta' \Upsilon_{m,0}'^< \frac{k_m \kappa_0}{k_0 \kappa_m} (\hat{n}_m'^< e^{-jk_m^< \cdot r}$$
$$+\,R_m'^< e^{+j2\kappa_m z_m} \hat{n}_m'^> e^{-jk_m^> \cdot r})$$
$$\cdot\,(\hat{n}_0'^< e^{+jk_0^< \cdot r'} + R_0'^> e^{-j2\kappa_0 z_0^+} \hat{n}_0'^> e^{+jk_0^> \cdot r'})], \quad (44)$$

where Δ is defined as (Ω/Λ), z_0^+, z_0^-, z_m are specified in Fig. 1, and $k_i^{\gtrless}$ is given by

$$k_i^{\gtrless} = \hat{x}k_x + \hat{y}k_y \pm \hat{z}\kappa_i; \quad |k_i^{\gtrless}| = k_i = \omega\sqrt{\mu_i \epsilon_i}. \quad (45)$$

and, unit vectors $\hat{n}''$, and $\hat{n}_i'$ are defined as

$$\hat{n}'' = \frac{\hat{x}k_y - \hat{y}k_z}{k_t}; \quad \hat{n}_i'^{\gtrless} = \frac{(-\hat{x}k_x - \hat{y}k_y)\kappa_i \pm \hat{z}k_t^2}{k_i k_t}. \quad (46)$$

Also note that $R_0^> = R^>(z_0^+)$, $R_0^< = R(z_0^-)$, and $R_m^{\gtrless} = R^{\gtrless}(z_m)$ for $m \gtrless 0$. The physical interpretation of the parameters defined here will be discussed in the following section.

V. Physical Interpretation of the Dyadic Green's Function for a Multilayered Media

In this section we will try to give some physical insight to the dyadic Green's function of the multilayered media derived in the preceding section.

The double prime, (″), denotes plane waves in the PWS representation for which the electric field is normal to the plane of incidence, (i.e., the plane defined by the propagation vector, k, and the direction normal $\hat{z}$); thus, the polarization of electric field vector, $\hat{n}''$, is given by

$$\hat{n}'' = \frac{-\hat{z} \times k}{|\hat{z} \times k|} = \frac{\hat{x}k_y - \hat{y}k_x}{k_t}. \quad (47)$$

Likewise the prime, (′), denotes plane waves with the electric field in the plane of incidence (with the magnetic field normal to the plane of incidence). In this case, the polarization of the electric field vector, $\hat{n}'$, is given by

$$\hat{n}'^{\gtrless} = \pm\frac{\hat{n}'' \times k^{\gtrless}}{|\hat{n}'' \times k|} = \frac{(-\hat{x}k_x - \hat{y}k_y)\kappa \pm \hat{z}k_t^2}{kk_t} \quad (48)$$

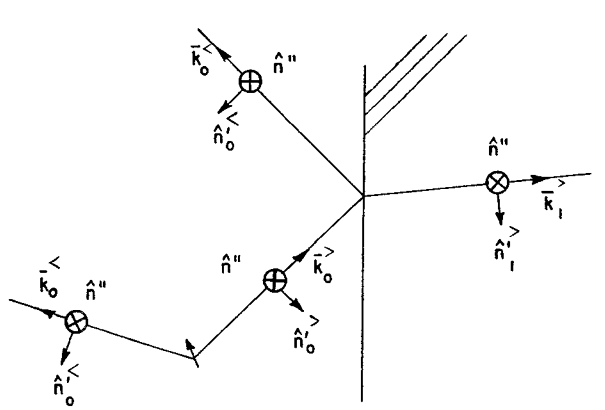


Fig. 3. Directions of $k_m^{\gtrless}$, $n_m'^{\gtrless}$ and n_m'' on either side of interface (0); ($m = 0, 1$).

where $\hat{n}'^{\gtrless}$ is for the wave traveling in the $\pm\hat{z}$ directions as shown in Fig. 3.

The dyadic Green's function evaluated in the region (m) consists of the spectrum of two types of plane waves excited by the source at $z = z'$ in region (0); these are the direct (incident) plus reflected waves. The total "effective" incident wave at $z = z_0^+$ is given by (see also (A11) of the Appendix),

$$e^{-j\kappa_0 z_0^+} \Delta (e^{jk_0^> \cdot r'} + R_0^< e^{j2\kappa_0 z_0^-} e^{jk_0^< \cdot r'});$$
$$\Delta = \frac{1}{1 - R_0^> R_0^< e^{-j2\kappa_0 d_0}}, \quad (49)$$

and Δ is the sum of the geometric series,

$$\Delta = 1 + \alpha + \alpha^2 + \alpha^3 + \cdots;$$
$$\alpha = R_0^>(z_0^+) R_0^<(z_0^-) e^{-j2\kappa_0 d_0}. \quad (50)$$

Physically Δ in (50) is the total sum of the plane waves traveling in $+\hat{z}$ or $-\hat{z}$ directions which result from the infinite number of bounces at the interfaces of slab (0), therefore it can be viewed as the "effective" incident wave at $z = z_0^+$, as is shown geometrically in Fig. 4.

The total incident wave at $z = z_0^+$ is transmitted through the slabs (0 to m), by the effective transmission coefficients, $\Upsilon_{m,0}^>$ (see (A12) of the Appendix),

$$\Upsilon_{m,0}^> = (T_0 e^{-j\kappa_1 d_1})(T_2 e^{-j\kappa_2 d_2}) \cdots (T_{m-1} e^{-j\kappa_m d_m}), \quad (51)$$

where d_i is thickness of the slab (i), (for $i = 0$ to m). At the slab m, the total field will be the superposition of the effective incident field plus the effective reflected field from the boundary at $z = z_m$ as shown in Fig. 5.

Note that the ratio of $k_m \kappa_0 / k_0 \kappa_m$ in the TM (′) part of (44) is simply the ratio of the cosine of the angles that k_0 and k_m make with the normal of the interface which is depicted in Fig. 6, and results from the continuity of the tangential TM electric fields at the each interface.

Although the limits of the spectral integral extend from $-\infty$ to ∞, the reflection and transmission coefficients, Γ_m and τ_m, for each interface and hence, the effective reflection and transmission coefficients, R_m and Υ_m, have an asymptotic limit for large value of k_t. Figs. 7–10 show the

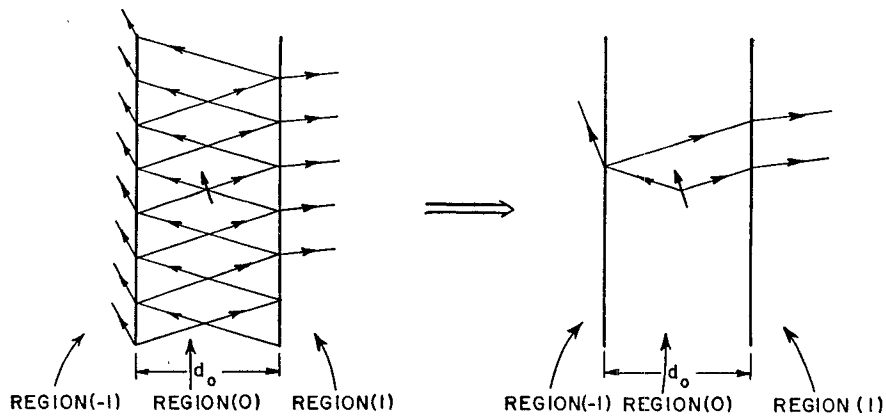


Fig. 4. Plane waves bouncing back and forth at the interfaces of the slab (0) and its equivalent representation.

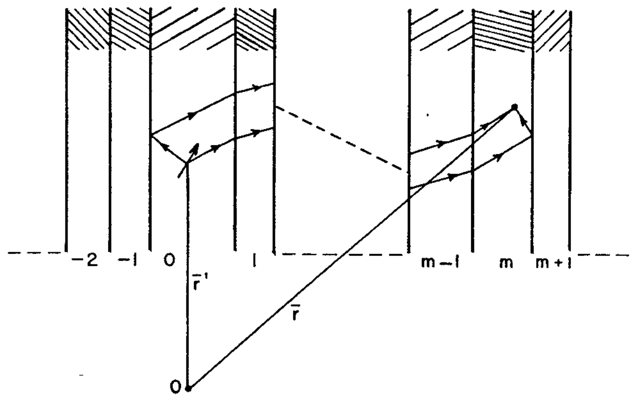


Fig. 5. Physical interpretation of incident and reflected waves in the slab m due to the point current dipole source in the slab (0).

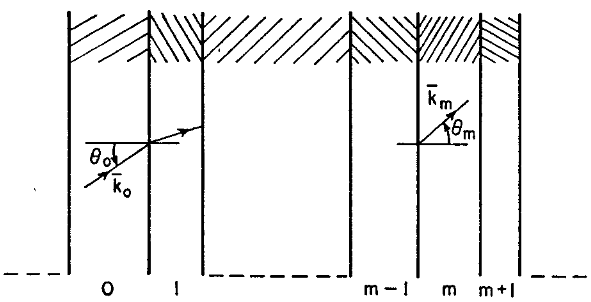


Fig. 6. Direction cosines that k_0 and k_m make with the normal $\hat{z}$, these result from the continuity of the tangential TM electric field at each interface; $\theta_0 = \cos^{-1}(\kappa_0/k_0)$, $\theta_m = \cos^{-1}(\kappa_m/k_m)$.

real and imaginary parts of effective reflection and transmission coefficients for one, two and three layer geometries as a function of normalized k_t, (with respect to the free space wave number, k_0), for TM and TE cases, respectively. It is evident that the values of these coefficients approach certain limiting constants for large values of k_t. It can be seen from Figs. 8 and 10 that the values of effective transmission coefficients approach zero for k_t larger than 3; physically this implies that no evanescent wave with a large transverse wave number k_t can penetrate through the layers. One can of course predict these phenomena by taking the limits of the reflection and transmission coefficients of (A12)–(A15) as k_t goes to infinity. That is,

$$R_m^{\gtreqless} \approx \Gamma^{\gtreqless}_{m|k_t >> k_m};$$

$$\lim_{k_t \to \infty} R_m^{\gtreqless} = \lim_{k_t \to \infty} \Gamma_m^{\gtreqless} \to \begin{cases} \dfrac{\epsilon_m - \epsilon_{m\pm 1}}{\epsilon_m + \epsilon_{m \pm 1}} & \text{for TM (')} \\ \dfrac{\mu_{m\pm 1} - \mu_m}{\mu_{m\pm 1} + \mu_m} & \text{for TE ('')} \end{cases} \tag{52}$$

and

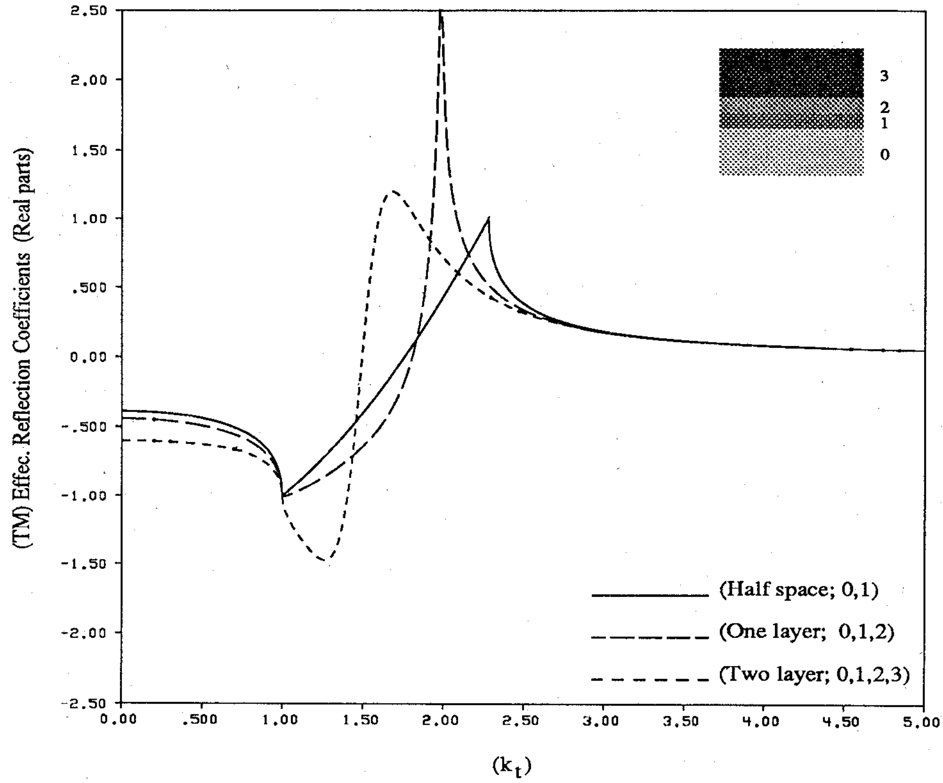
$$T_m^{\gtreqless} \approx \tau^{\gtreqless}_{m|k_t >> k_m};$$

$$\lim_{k_t \to \infty} T_m^{\gtreqless} = \lim_{k_t \to \infty} \tau_m^{\gtreqless} \to \begin{cases} \dfrac{2\epsilon_m}{\epsilon_m + \epsilon_{m\pm 1}} & \text{for TM (')} \\ \dfrac{2\mu_{m\pm 1}}{\mu_{m\pm 1} + \mu_m} & \text{for TE ('').} \end{cases} \tag{53}$$

As is evident from (51) for any multilayered media with a nonzero thickness, we will have

$$\lim_{k_t \to \infty} \Upsilon^{\gtreqless}_{m,0} \longrightarrow 0. \tag{54}$$

The numerical implication of this phenomena is that for large values of k_t, the effective reflection and transmission coefficients, R_m and T_m, can be replaced by their associated half-space reflection and transmission coefficients, Γ_m and τ_m. Also it is evident from Figs. 7 and 8 that for a set of constitutive parameters and layer thicknesses, there exist some value of k_t for which the denominators of effective reflection and transmission coefficients go to zero and consequently these coefficients become singular. These values of k_t correspond to the surface wave modes, and the associated residues are proportional to the fields of these modes where are launched by the impressed source [6], [20]. Also, the sharp variation of these effective reflection and transmission coefficients, at the various points



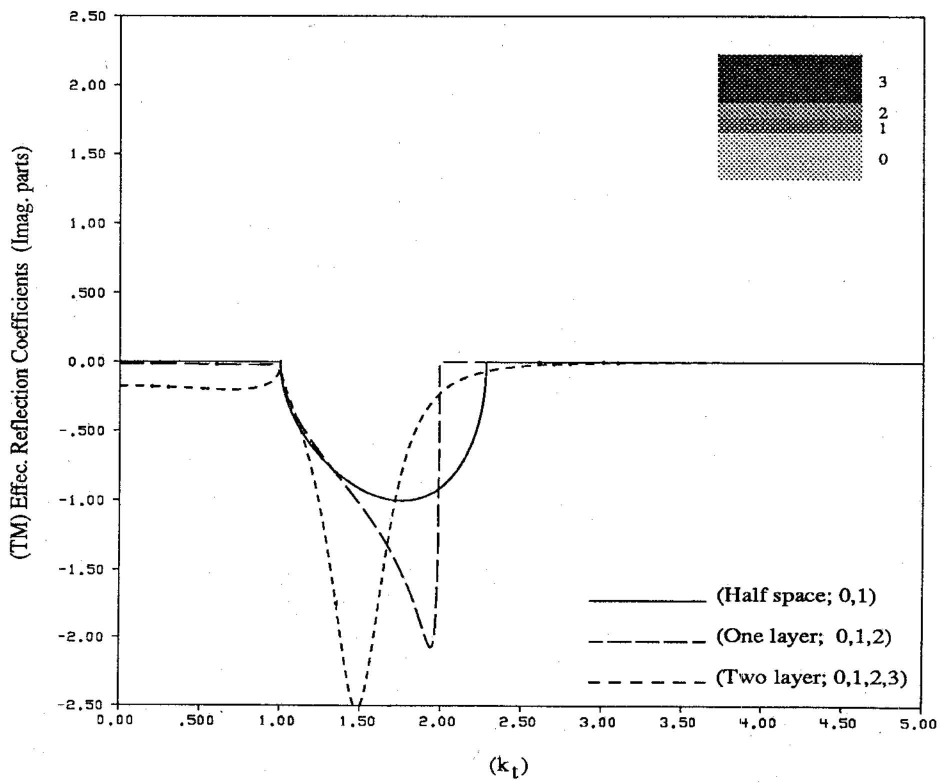
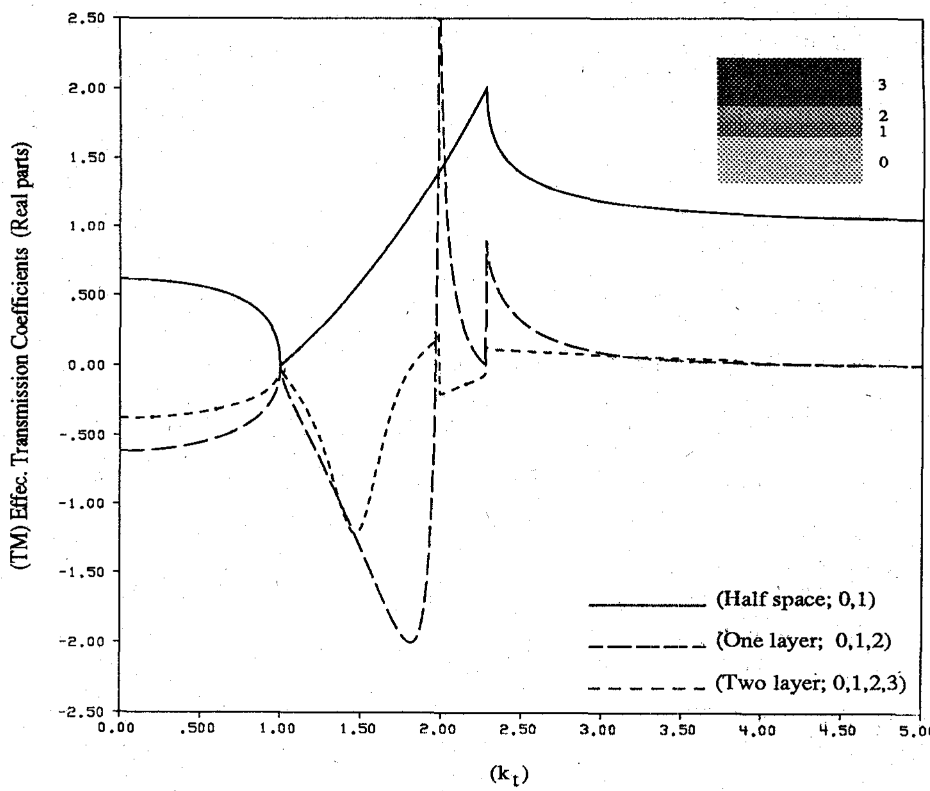


Fig. 7. Real and imaginary parts of effective TM reflection coefficients as a function of normalized k_t (with respect to k_0) for a half-space, as well as for one-layer and two-layer media on a half-space. The relative constitutive parameters and layer thicknesses are: ($\mu_{0,r} = 1.0$, $\epsilon_{0,r} = 1.0$), ($\mu_{1,r} = 1.2$, $\epsilon_{1,r} = 3.25$), ($\mu_{2,r} = 1.3$, $\epsilon_{2,r} = 10.2$), ($\mu_{3,r} = 1.6$, $\epsilon_{3,r} = 2.2$), ($d_1/\lambda_0 = 0.1$), and ($d_2/\lambda_0 = 0.1$).



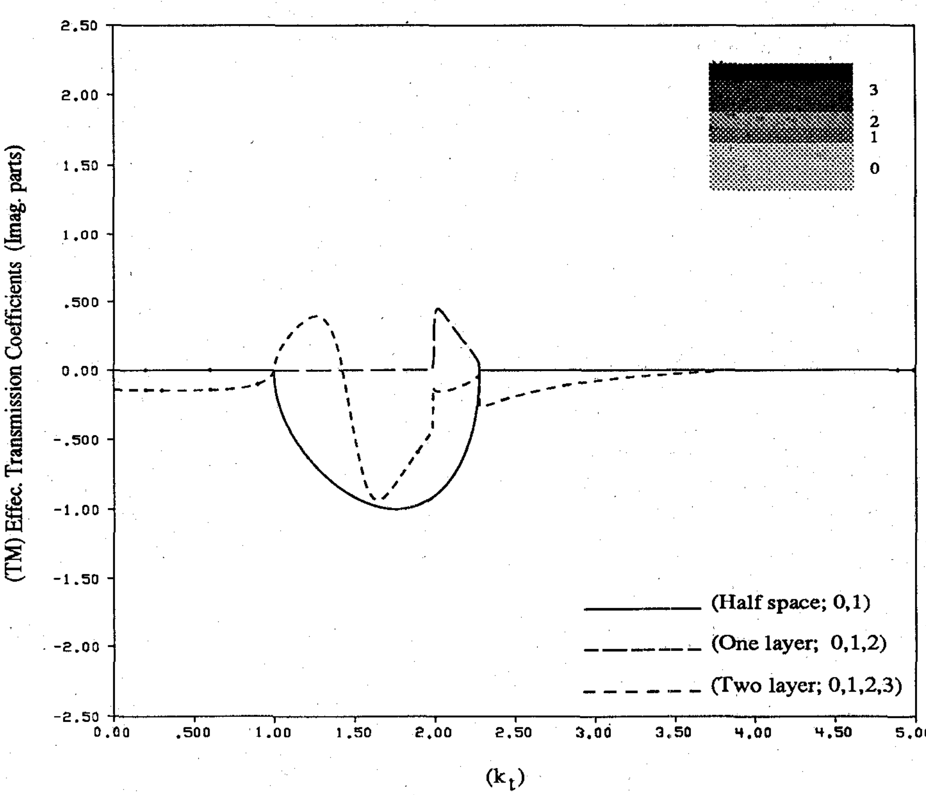


Fig. 8. Real and imaginary parts of effective TM transmission coefficients as a function of normalized k_t (with respect to k_0) for a half-space, as well as for one-layer and two-layer media on a half-space. The relative constitutive parameters and layer thicknesses are: ($\mu_{0,r} = 1.0$, $\epsilon_{0,r} = 1.0$), ($\mu_{1,r} = 1.2$, $\epsilon_{1,r} = 3.25$), ($\mu_{2,r} = 1.3$, $\epsilon_{2,r} = 10.2$), ($\mu_{3,r} = 1.6$, $\epsilon_{3,r} = 2.2$), ($d_1/\lambda_0 = 0.1$), and ($d_2/\lambda_0 = 0.1$).

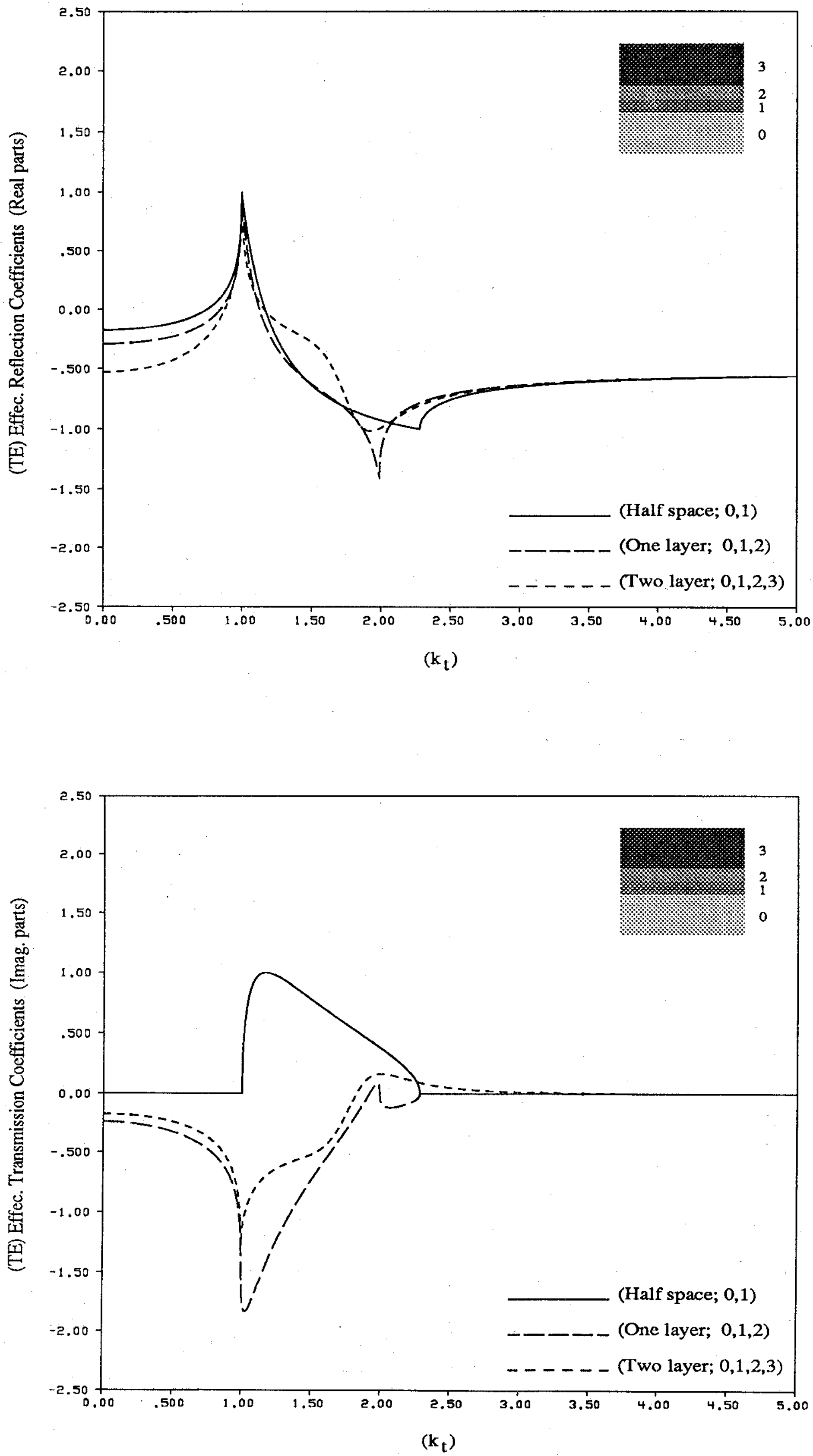


Fig. 9. Real and imaginary parts of effective TE reflection coefficients as a function of normalized k_t (with respect to k_0) for a half-space, as well as for one-layer and two-layer media on a half-space. The relative constitutive parameters and layer thicknesses are: ($\mu_{0,r} = 1.0$, $\epsilon_{0,r} = 1.0$), ($\mu_{1,r} = 1.2$, $\epsilon_{1,r} = 3.25$), ($\mu_{2,r} = 1.3$, $\epsilon_{2,r} = 10.2$), ($\mu_{3,r} = 1.6$, $\epsilon_{3,r} = 2.2$), ($d_1/\lambda_0 = 0.1$), and ($d_2/\lambda_0 = 0.1$).

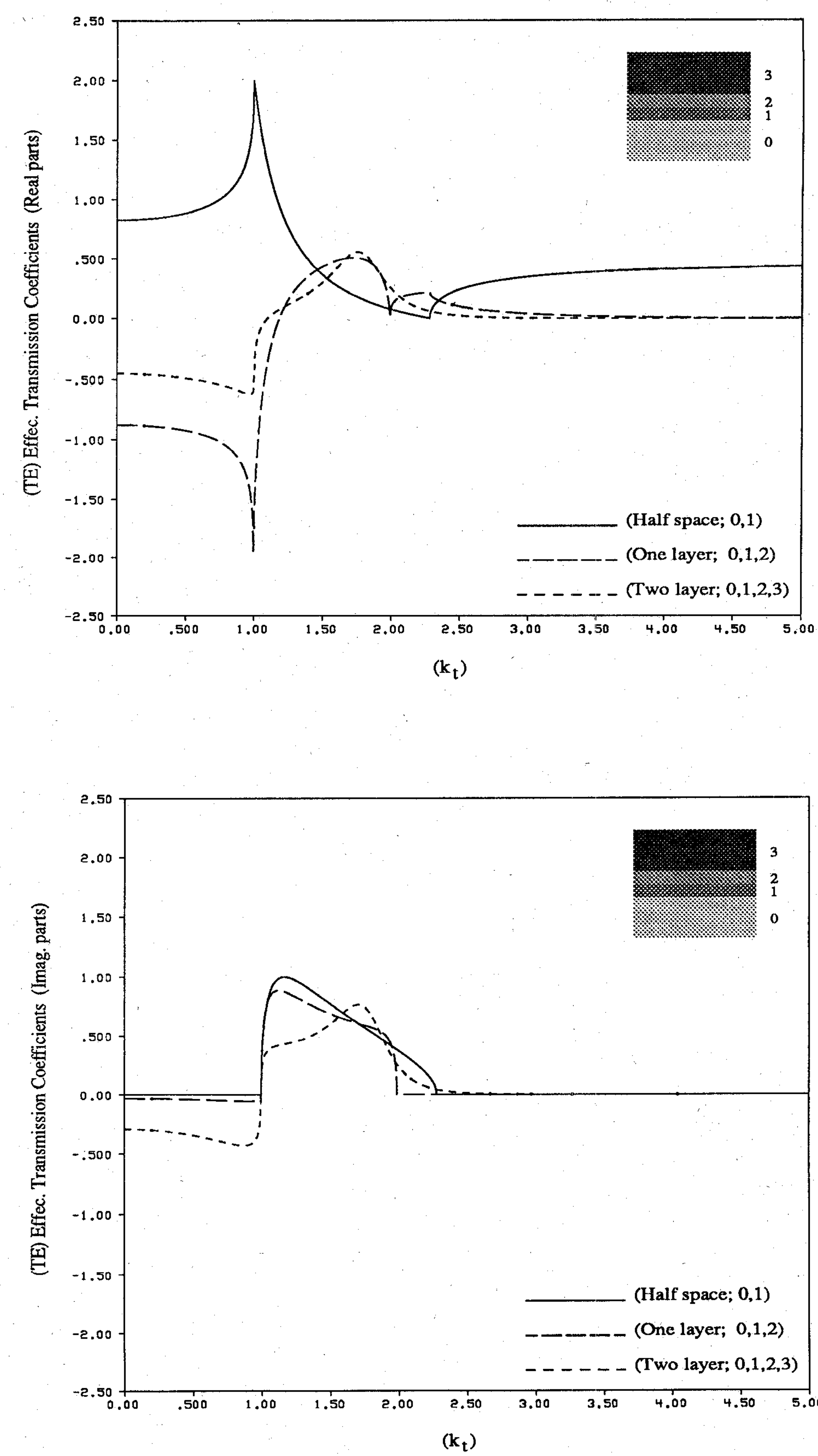


Fig. 10. Real and imaginary parts of effective TE transmission coefficients as a function of normalized k_t (with respect to k_0) for a half-space, as well as for one-layer and two-layer media on a half-space. The relative constitutive parameters and layer thicknesses are: ($\mu_{0,r} = 1.0$, $\epsilon_{0,r} = 1.0$), ($\mu_{1,r} = 1.2$, $\epsilon_{1,r} = 3.25$), ($\mu_{2,r} = 1.3$, $\epsilon_{2,r} = 10.2$), ($\mu_{3,r} = 1.6$, $\epsilon_{3,r} = 2.2$), ($d_1/\lambda_0 = 0.1$), and ($d_2/\lambda_0 = 0.1$).

in these Figures result from an abrupt phase change of the associated half-space reflection and transmission coefficients at the vicinity of $k_t = k_m$, for (m = 0, 1, 2, or 3).

VI. Conclusion

A relatively simple and systematic approach is taken to drive the dyadic Green's function for a multilayered dielectric/magnetic media via the two ($\hat{z}$)-directed solenoidal eigenfunctions, and the utilization of the Lorentz reciprocity theorem such that it provides a useful physical interpretation. It is shown that the Green's dyadic can be written in terms of the spectrum of plane waves (TE and TM) which resemble the response of a source excited multiconnected piece-wise uniform transmission line. The concept of effective reflection and transmission coefficients is discussed, and the physical interpretation of the individual terms along with the limiting behavior of some of these terms is given.

Appendix
Piecewise Uniform Transmission Line Theory

In this Appendix we briefly review the piecewise uniform transmission line theory. As explained earlier, the z and z' functional dependence of the field quantities excited by a electric point dipole current source in a general multilayered media is analogous to the problem of source excitation of a piecewise uniform transmission line. The voltage and current on a source free uniform transmission line with wave number κ_m and characteristic impedance η_m can be expressed as

$$V_m(z) = V_{inc,m}(z_0)(e^{-j\kappa_m(z-z_0)} + R_m(z_0)e^{j\kappa_m(z-z_0)}),$$

$$I_m(z) = \frac{V_{inc,m}(z_0)}{\eta_m}(e^{-j\kappa_m(z-z_0)} - R_m(z_0)e^{j\kappa_m(z-z_0)}), \quad \text{(A1)}$$

where $V_{inc,m}(z_0)$ and $R_m(z_0)$ are the incident voltage and reflection coefficient respectively at point $z = z_0$. The reflection coefficient, $R_m(z)$, and the impedance, $Z_m(z)$, at a point z are related by

$$R_m(z) = \frac{Z_m(z) - \eta_m}{Z_m(z) + \eta_m}; \quad Z_m(z) = \frac{V_m(z)}{I_m(z)}. \quad \text{(A2)}$$

It is desired to derive some expressions for a piecewise uniform transmission line that relate the voltages and currents at a pair of points on the line which are located in different sections. Let us first consider a simple configuration shown in Fig. 11 which consists of two semi-infinite transmission lines corresponding to regions ($n - 1$) and ($n + 1$), connected with a finite line, $d_n = z_n - z_{n-1}$, corresponding to region (n). For a known incident voltage in region ($n - 1$), it is of interest to find voltages and currents in different sections of the transmission line. For doing so, one needs to find the incident voltage and reflection coefficient in each region. The reflection coefficient at z_{n-1} in region ($n - 1$) can be written as

$$R_{n-1} = \frac{Z(z_{n-1}) - \eta_{n-1}}{Z(z_{n-1}) + \eta_{n-1}}, \quad \text{(A3)}$$

where

$$Z(z_{n-1}) = \eta_n \frac{1 + R_n e^{-j2\kappa_n d_n}}{1 - R_n e^{-j2\kappa_n d_n}}; \quad R_n = \frac{\eta_{n+1} - \eta_n}{\eta_{n+1} + \eta_n}. \quad \text{(A4)}$$

After incorporating (A4) into (A3), the reflection coefficient R_{n-1} can be expressed as

$$R_{n-1} = \frac{\Gamma_{n-1} + R_n e^{-j2\kappa_n d_n}}{1 + \Gamma_{n-1} R_n e^{-j2\kappa_n d_n}}; \quad \Gamma_{n-1} = \frac{\eta_n - \eta_{n-1}}{\eta_n + \eta_{n-1}}. \quad \text{(A5)}$$

Expression R_{n-1} in (A5) is called "effective" reflection coefficient for region ($n - 1$). It is a coefficient that relates all interactions from the presence of other regions to the incident voltage in region ($n - 1$). One can also relate the incident voltages of regions ($n - 1$) and (n) in the following form:

$$\begin{aligned} V_{n-1}(z_{n-1}) &= V_{inc,n-1}(z_{n-1})(1 + R_{n-1}(z_{n-1})) \\ &= V_{inc,n}(z_{n-1})(1 + R_n e^{-j2\kappa_n d_n})), \end{aligned} \quad \text{(A6)}$$

hence; $V_{inc,n}(z_{n-1})$ can be expressed in terms of $V_{inc,n-1}(z_{n-1})$ as

$$V_{inc,n}(z_{n-1}) = T_{n-1}(z_{n-1})V_{inc,n-1}(z_{n-1}), \quad \text{(A7)}$$

where

$$T_{n-1} = \frac{1 + R_{n-1}}{1 + R_n e^{-j2\kappa_n d_n}}. \quad \text{(A8)}$$

After substituting (A5) for R_{n-1} in (A8), T_{n-1} can be expressed as

$$T_{n-1} = \frac{\tau_{n-1}}{1 + \Gamma_{n-1} R_n e^{-j2\kappa_n d_n}}; \quad \tau_{n-1} = 1 + \Gamma_{n-1}. \quad \text{(A9)}$$

T_{n-1} in (A9) is called "effective" transmission coefficient. It is a coefficient that relates the incident wave of region ($n - 1$) to the incident wave of region (n). Therefore, the voltage at a point z in region (n) can be expressed in terms of the incident voltage at point z_0 in region ($n - 1$) by incorporating (A7) into (A1); hence,

$$\begin{aligned} V_n(z) = {} & V_{inc,n-1}(z_0)e^{-j\kappa_{n-1}(z_{n-1}-z_0)}T_{n-1}e^{-j\kappa_n d_n} \\ & \cdot (e^{-j\kappa_n(z-z_n)} + R_n e^{+j\kappa_n(z-z_n)}). \end{aligned} \quad \text{(A10)}$$

The incident voltage in region ($n + 1$) can likewise be found in terms of the voltage in region (n).

This formulation can be generalized to the total of ($N + 2$) number of finite length transmission lines, (0^{th} and $(N + 1)^{th}$ regions are semi-infinite), with the characteristic impedance and wavenumber of η_m and κ_m, respectively for ($0 \le m \le N + 1$), as shown in Fig. 12. The

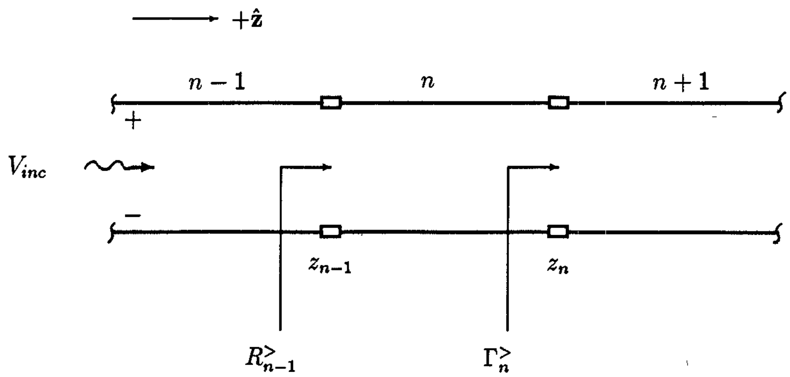


Fig. 11. Two infinite transmission lines connected with a finite length transmission line at $z = z_{n-1}$, and $z = z_n$.

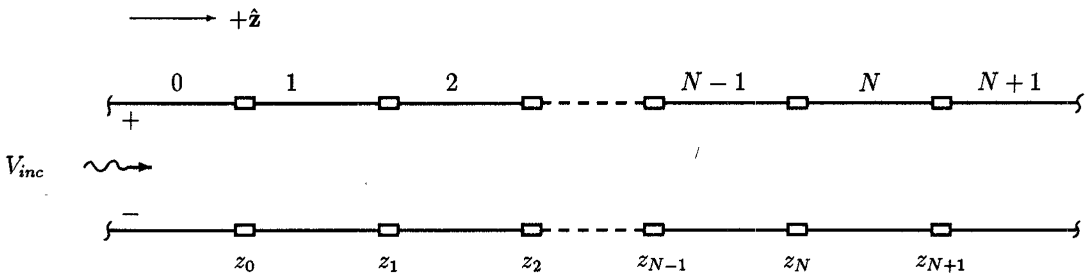


Fig. 12. General piecewise uniform transmission line; incident waves travel in $+\hat{z}$-direction.

voltage and current at point z in region (m), as a function of the incident voltage at point $z = 0$ in region (0) can be written as

$$V_m(z) = V^>_{inc,0}(0)e^{-j\kappa_0 z_0}\Upsilon^>_{m,0}(e^{-j\kappa_m(z-z_m)} + R^>_m e^{+j\kappa_m(z-z_m)}),$$

$$I_m(z) = \frac{V^>_{inc,0}(0)}{\eta_m} e^{-j\kappa_0 z_0}\Upsilon^>_{m,0}(e^{-j\kappa_m(z-z_m)} - R^>_m e^{+j\kappa_m(z-z_m)}), \quad \text{(A11)}$$

where, $\Upsilon^>_{m,0}$ and $R^>_m$ are respectively defined as

$$\Upsilon^>_{m,0} = \prod_{i=0}^{m-1} T^>_i(z_i)e^{-j\kappa_{i+1}d_{i+1}};$$

$$T^>_i(z_i) = \frac{\tau^>_i}{1 + \Gamma^>_i R^>_{i+1}e^{-j2\kappa_{i+1}d_{i+1}}}, \quad \text{(A12)}$$

and

$$R^>_m(z_m) = \frac{\Gamma^>_m + R^>_{m+1}e^{-j2\kappa_{m+1}d_{m+1}}}{1 + \Gamma^>_m R^>_{m+1}e^{-j2\kappa_{m+1}d_{m+1}}}, \quad \text{(A13)}$$

where $R^>_{m+1}$ and $T^>_i$ can be calculated by successive applications of (A5) and (A9), starting from region N. The superscript $(>)$ explicitly used to imply that the incident field travels in $(+\hat{z})$-direction.

All equations derived here are applicable for the case in which the incident field travels in $(-\hat{z})$-direction, provided $\kappa_m \rightarrow -\kappa_m$ and $(m\mp) \rightarrow (m\pm)$. Hence; the effective reflection and transmission coefficients for the geometry depicted in Fig. 13 are respectively defined as

$$R^<_m(z_m) = \frac{\Gamma^<_m + R^<_{m-1}e^{-j2\kappa_{m-1}d_{m-1}}}{1 + \Gamma^<_m R^<_{m-1}e^{-j2\kappa_{m-1}d_{m-1}}};$$

$$\Gamma^<_m = \frac{\eta_{m-1} - \eta_m}{\eta_{m-1} + \eta_m}, \quad \text{(A14)}$$

$$T^<_m(z_m) = \frac{\tau^<_m}{1 + \Gamma^<_m R^<_{m-1}e^{-j2\kappa_{m-1}d_{m-1}}};$$

$$\tau^<_m = 1 + \Gamma^<_m. \quad \text{(A15)}$$

and

$$\Upsilon^<_{m,0} = \prod_{i=0}^{m+1} T^<_i(z_i)e^{-j\kappa_{i-1}d_{i-1}}. \quad \text{(A16)}$$

Note that in this case the subscript m of the indices of the layers in Fig. 13 is monotonically decreasing; (i.e., $m \leq 0$; $z_{m-1} < z_m$).

It is evident from the above analysis that once the incident waves on either side of the source in region (0) are known, the voltages and currents of other regions of the transmission line will be specified.

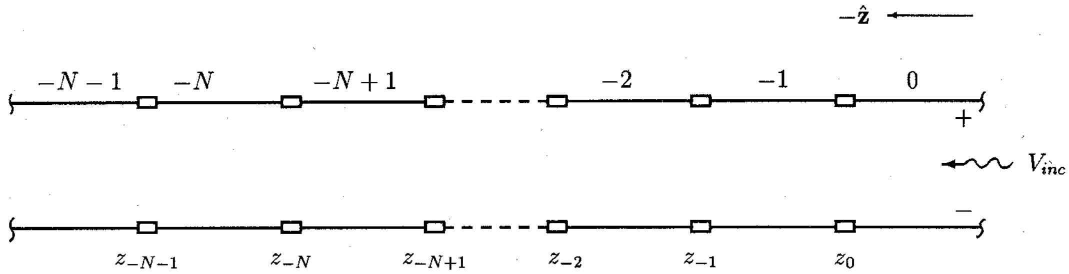


Fig. 13. General piecewise uniform transmission line; incident waves travel in $-\hat{z}$-direction.

REFERENCES

[1] A. Sommerfeld, *Partial Differential Equations in Physics*. New York: Academic, 1949.

[2] J. R. Wait, *Antenna Theory*, part 2, Collin and Zucker Eds., New York: McGraw-Hill, 1969, pp. 438-506.

[3] A. Banos, *Dipole Radiation in the Presence of a Conducting Half Space*. New York: Pergamon, 1966.

[4] C. H. Stoyer, "Electromagnetic fields of dipoles in stratified media," *IEEE Trans. Antennas Propag.*, pp. 547-552, July 1977.

[5] J. R. Wait, *Electromagnetic Wave Theory*. New York: Harper & Row, 1985.

[6] L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves*. Englewood Cliffs, NJ: Prentice-Hall, 1973, pp. 442-630.

[7] J. A. Kong, Ed., *Research Topics in Electromagnetic Wave Theory*. New York: Wiley-Interscience, 1981, pp. 210-234.

[8] J. A. Kong, *Theory of Electromagnetic Waves*, 2nd ed., New York: Wiley, 1986.

[9] S. Barkeshli and P. H. Pathak, "On the dyadic Green's function of the multilayered dielectric/ferrite media," in *Proc. IEEE Int. AP-S Symp. and National Radio Science Meeting*, Blacksburg, VA, June 15-19, 1987.

[10] —, "On the dyadic Green's function of an infinite periodic array of arbitrary oriented dipole in a multi-layered media," in *Proc. IEEE Int. AP-S Symp. and National Radio Science Meeting*, Blacksburg, VA, June 15-19, 1987.

[11] J. S. Bagby and D. P. Nyquist, "Dyadic Green's function for integral electronic and optical circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, no. 2, pp. 206-210, Feb. 1987.

[12] M. S. Viola and D. P. Nyquist, "An observation on the Sommerfeld-integral representation of the electric dyadic Green's function for layered media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-36, pp. 1289-1292, Aug. 1988.

[13] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960.

[14] P. H. Pathak, "On the eigenfunction expansion of electromagnetic dyadic Green's functions," *IEEE Trans. Antennas Propagat.*, vol. AP-31, pp. 837-846, 1983.

[15] C. T. Tai, *Dyadic Green's Function in Electromagnetic Theory*. Scranton, PA: Intext, 1971.

[16] J. Van Bladel, *Electromagnetic Fields*. New York: McGraw-Hill, 1964, pp. 537-545.

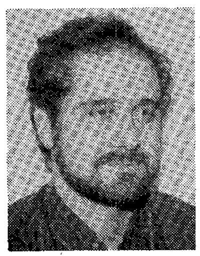
[17] J. R. Wait, *Geo-Electromagnetism*. New York: Academic, 1982.

[18] B. A. Munk, G. A. Burrell, T. W. Kornbau, "A general theory of periodic surfaces in stratified dielectric media," Tech. Rep. 784346-2, ElectroScience Laboratory, Department of Electrical Engineering, The Ohio State University, Columbus, Nov. 1977.

[19] B. A. Munk, "A general theory of periodic surfaces in a Stratified dielectric media," Tech. Rep. 715582-4, ElectroScience Laboratory, Dept. of Electrical Engineering, The Ohio State University, Columbus, Feb., 1986.

[20] S. Barkeshli and P. H. Pathak, "Radial propagation and steepest descent path representation of the planar microstrip dyadic Green's function," *Radio Science*, vol. 25, no. 2, pp. 161-174, Mar.-Apr. 1990.

[21] W. C. Chew, *Waves and Fields in Inhomogeneous Media*. New York: Van Nostrand Reinhold, 1990.



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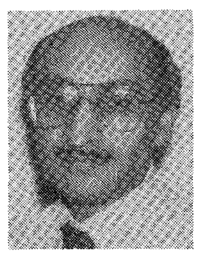

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